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Optimal Portfolio Analysis Using Markowitz Model and Single Index Model

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Abstract - This study aims to determine the composition of the optimal portfolio formation results of the Markowitz Model and Single Index Model. This study also aims to compare the results of optimal portfolio formation and performance of the Markowitz Model and Single Index Model. The population in this study are all stocks included in the LQ45 index listed on the Indonesia Stock Exchange for the period 2018-2022. The research sample is stocks that are consistently listed in the LQ-45 index during the 2018-2022 period. The number of research samples based on these conditions was 27 stocks. The results of this study are: In the Markowitz Model, 5 optimal portfolio-forming stocks are obtained, namely ADRO (49%), ANTM (22%), BBCA (7%), ICBP (5%), and KLBF (16%). With a portfolio return rate of 0.025382 (2.54%) and a portfolio risk of 0.081342 (8.13%). In the Single Index Model, 9 optimal portfolio-forming stocks are obtained, namely UNTR (15.18%), PTBA (3.38%), MNCN (4.15%), ICBP (35.59%), EXCL (6.62%), BBTN (0.92%), BBCA (32.55%), ANTM (0.04%) and ADRO (1.56%). With a portfolio return rate of 0.02066 (2.1%) and a portfolio risk of 0.04005 (4%). The results of the independent sample t-test processing of the return show that there is no difference between the Markowitz Model return and the Single Index Model return.

Keywords: Optimal Portfolio; Markowitz Model; Single Index Model

1. Introduction

Indonesian economy does not free from influence dynamics global economy. In 2020, the global economy experienced growth slow economy and uncertainty. Growth slow economy caused by pressure strong inflation, tightening response policy monetary, policy controlling Covid-19 in China. Things the has cause trend growth Indonesia's economy follows suit slowing down (www.bi.go.id). The Indonesian economy in 2020 experienced contraction growth of 2.07% (c-to-c) compared in 2019. The Indonesian economy will grow in 2021 of 3.69%, more tall compared achievements 2020 experienced decline growth of 2.07%. Temporary Therefore, the Indonesian economy will grow in 2022 of 5.31%, more tall compared achievements 2021 (bps.go.id).

IHSG in 2020 experienced decline of -5.08%, result from exists pandemic. IHSG in 2021 experienced enhancement amounting to 10.07% along with getting better global economy. In 2022, IHSG will experience a decrease that does not too Far from 2021 that is of 4.08%.

Development investment shares in Indonesia for the 2020-2022 period are listed in Table 1 below this.

Table 1. Development Stock Investment in Indonesia for the 2020-2022 Period

Year	Share Volume (million share)	Share Value (million rupiah)	IHSG	Growth Rate
2020	523 856.00	350 131.00	5 979.07	-0.0508
2021	524 544.00	271 478.00	6 581.48	0.1007
2022	536 080.00	289 705.00	6,850.62	0.0408

Source : ojk.go.id and idx.co.id

When building portfolio, a rational investor will choose possible investment give profit maximum with risk specific, depending on the preferences of each investor. According to (Hartono, 2020) rational investors will choose constituents portfolio with optimize one from two aspect namely return and risk portfolio, that is portfolio effective investment. Efficient portfolio is portfolio that offers largest expected return For something risk certain or portfolio with risk smallest For certain return expectations (Hartono, 2020). The optimal portfolio is portfolio that investors choose from a number of available options in gathering portfolio efficient (Tandelilin, 2017). In finance, especially in empirical asset pricing, the trade-off between the returns and the risk of a portfolio is one of the key issues. The Sharpe Ratio is an important way to quantify such trade-off. (Caner et al, 2023)

Determination optimal portfolio can do with Lots way, like build containing portfolio Lots asset, build portfolio in a way random, and do diversification use Markowitz Model and Single Index Model. The Markowitz model was developed by Markowitz in 1952 can overcome weakness from diversification shares held in a way random. Assumption that addition amount shares in One portfolio in a way Keep going continuously will give increasing benefits big, different with the Markowitz Model. This model believes that addition share in a way Keep going continuously on one portfolio , on one point certain will the more reduce benefit diversification The single index model was developed by William Sharpe (Sharpe, 1963), (Sharpe, 1992), (Sharpe, 1994) own characteristics that is use assumption that securities move together No Because off - market effects (eg effect from industry or companies That itself), but rather Because have general relationship to market (Hartono, 2020) Single index models are also possible used For calculate expected return and risk portfolio.

On research previously carried out by (Yuana et al., 2016) with title " Analysis Formation Optimal Stock Portfolio Using the Markowitz Model as the Determination Basis Investment ." With sample shares listed on the Jakarta Islamic Index for the period June 2013-November 2015. Research that get results that based on the Markowitz model is obtained seven (7) shares namely, UNVR of 31.23%, UNTR of 24.15%, KLBF of 14.78%, ICBP of 10.93%, AKRA of 10.41%, AALI of 6.79%, and WIKA of 1.71 %. Providing portfolio returns of 0.58% and risk portfolio of 0.12%. Whereas study about single index model in formation optimal portfolio ever conducted by (Shinta et al., 2016) with results study show of 5 shares from 31 samples share including optimal portfolio . Shares shaper optimal portfolio along with composition: AALI (1.15%), UNVR (5.36%), EMTK (41.00%), HMSP (29.79%), and ICBP (22.70%). Formed portfolio produce expected return portfolio amounted to 2.71% and level risk portfolio of 0.14%.

Then study previously done (Ratna et al., 2016) obtain expected return results portfolio that will obtained by investors in 2012 were of 3.03% with risk of 0.04%, the expected return in 2013 is 1.93 % with risk amounted to 0.06%, the expected return in 2014 was 3.38% with risk of 0.02%. Study previously carried out by (Hidayatul et al., 2017) with title " Comparative Analysis Optimal Portfolio Using Single Index Model and Markowitz Model on JII Shares." With sample shares listed on the Jakarta Islamic Index 2015-2016 period get results study that with the Markowitz model obtained the average portfolio return of 0.0104 and average risk portfolio of 0.02825. Huang (2024) research results point to an inverse association between the HHI and investment performance, suggesting that diversification tends to reduce investment risk and increase investment performance. Using forecasted risk measures to optimise portfolios does not yield statistically significant improvements in portfolio performance (Surtee et al, 2023), (Wiharno et al, 2023), (Lorimer et al, 2024).

This research is a development of previous research and the focus of this research is on forming an optimal portfolio using the Markowitz Model and Single Index Model . Researchers conducted an analysis of the LQ-45 stock index because by using LQ-45 shares as the research object, it was possible to find out what shares were actively traded on the Indonesian Stock Exchange and LQ-45 shares were the leading shares selected from each industrial sector. LQ-45 index shares are a group of companies with shares that tend to be stable because they have a high level of liquidity. The LQ-45 index shares are filled by 45 entities which are then selected according to criteria set by the Indonesian Stock Exchange and share turnover is carried out every six months.

Based on the background above, the researcher formulated the problem as follows:

1) What shares can form an optimal portfolio on the LQ-45 stock index listed on the IDX using the Markowitz Model?
2) What shares can form an optimal portfolio on the LQ-45 stock index listed on the IDX use Single Index Model?
3) What is the return and risk of the optimal portfolio formed on LQ-45 index shares listed on the Indonesian Stock Exchange?
4) What is the proportion of each share formed in the optimal portfolio of LQ-45 index shares listed on the IDX?
5) Which model between Markowitz Model and Single Index Model are better its performance in formation optimal portfolio?
6) is there is difference return and risk portfolio between Markowitz Model and Single Index Model based on analysis statistics?

Based on the problem formulation above, the objectives of this research are:

1) To find out which shares can form an optimal portfolio, use the LQ-45 stock index listed on the IDX using the Markowitz Model .
2) To find out which shares can form an optimal portfolio, use the LQ-45 stock index listed on the IDX using the Single Index Model .

When conditions start to improve, blue chip stocks are the first to be targeted by Investors (Suarjana & Surasmi, 2024). However, according to (Maulana, 2020) Investment decisions must have a relevant basis in order to achieve the goal of maximizing profits and minimizing risk, where this investment decision can be made by two parties, namely investors or investment managers. Investors or investment managers who invest in shares in the capital market are important to consider several factors, including the amount of capital to be invested, the investment period, the level of risk that will arise, and the amount of return that will be obtained (Maulana et al, 2024).

According to Jones (2014), "A portfolio with the highest level of expected return for a given level of risk before a portfolio with the lowest risk for a given level of expected return". This theory explains that the portfolio with the highest level of expected return for a given level of risk-return or the portfolio with the lowest risk for a certain level of expected return.

According to Husnan (2015), an efficient portfolio is a portfolio that produces the same level of profit with lower risk, or with the same risk providing a higher level of profit. An efficient portfolio is a portfolio that provides the largest expected return with a certain risk and provides the smallest risk with a certain expectation.

According to Hartono (2020), an efficient portfolio is a good portfolio, but not the best. An efficient portfolio only has one good factor, namely the expected return or risk factor. The optimal portfolio is the portfolio that investors choose from the many choices available in the efficient portfolio collection.

A portfolio is said to be efficient or optimal if the portfolio when compared with other portfolios meets the following conditions:
a. Provides the greatest ER (Expected Return) with the same risk.
b. Provides the smallest risk with the same ER.

A number of portfolios that meet these two conditions can be put into the efficient set or efficient frontier category. The Efficient Frontier is a line that shows a number of efficient portfolios, and all portfolios below this line are declared inefficient.

Based on the opinions of the experts above, it can be concluded that the optimal portfolio is a set of portfolios that investors will choose because they have the best expected return and risk.

One of the risks faced by investors is unsystematic risk. Diversification (portfolio) means that investors need to form a portfolio by selecting a combination of a number of assets in such a way that risk can be minimized without reducing expected returns (Tandelilin, 2017). According to (Tandelilin, 2017), there are two principles of diversification, namely:

1. Random Diversification
Random diversification is known as naïve diversification. This diversification occurs when investors invest their funds randomly in various different types of shares and hope that the return variance as a measure of portfolio risk will be reduced. Investors who carry out random diversification assume that the more types of assets they include in their portfolio, the greater the risk reduction benefits they will obtain.

2. Markowitz Diversification
Markowitz stated that what is very important in portfolio diversification is "Don't put all your eggs in one basket, because if that basket falls then all the eggs in the basket will break." In the context of investment, this teaching can be interpreted as "Don't invest all the funds we have in just one asset, because if that asset fails then all the funds we have invested will disappear." Markowitz's contribution is that in calculating portfolio risk it is not justified to simply add up all the asset risks in the portfolio, but must be calculated from the contribution of asset risk to the portfolio risk or what is called covariance. Covariance is an absolute measure that shows the extent to which the returns of two securities in a portfolio tend to move simultaneously.

Portfolio formation to reduce systematic risk cannot only be done with random diversification and Markowitz diversification. Investors can also use the single index model method to form a portfolio.
Based on the explanation above, it can be concluded that diversification is a strategy that can be used by combining investments in a portfolio to minimize the risks faced by investors.

a. Markowitz Model

There is a rule that states that investors should diversify their funds into several securities that provide the maximum expected return. We can allow the anticipated return value to include the risk value that is still allowed. Or utilize the return value of several types of securities including their risks (Markowitz, 1952).

According to (Tandelilin, 2017) an efficient portfolio is a portfolio that has the highest return value and has risk at a certain level or vice versa. One good factor that only an efficient portfolio has is the expected return or risk factor (Tania et al., 2018). Meanwhile, according to (Tandelilin, 2017), an optimal portfolio is a portfolio that matches the investor's chosen risk appetite from a collection of efficient portfolios. Both those owned by institutions and individuals (Ibnas et al., 2017).
For the Markowitz method of calculation that is :

1) Calculating Expected Return Value (E(Ri))

$$\mathrm{E(Ri)} = \frac{\Sigma \mathrm{Ri}}{\mathrm{n}}$$

Information:
E(Ri) = Expected return investment share i
n = Amount share

2) Calculating Stock Risk ($\boldsymbol{\sigma^2 i}$)

$$\sigma^2\mathrm{i} = \frac{\sqrt{\Sigma(\mathrm{Rit} - \mathrm{E(Ri)})^2}}{(\mathrm{n}-1)}$$

Information:
σ^2i = Variance of investment in stock i
Rit = Return from investment in stock i at condition t
E(Ri) = Expected return from investment share i

b. Single Index Model

(Sharpe, 1994) developed the second stage of the three stages of portfolio analysis contained in Markowitz's research that is Single Index Model . Shows a simple model of the relationship between securities, shows a simpler way of portfolio analysis, and provides evidence for the practical application of the Markowitz model.

There are several things that can differentiate between the Markowitz model and the Single Index Model (Suteja & Gunardi, 2016) :

1) Markowitz Model:
   - There are 3 basic assumptions, namely: a single investment period of one year, there are no transaction costs, and investor choices are based on expected return and risk.
   - Does not take into account the possibility of investors to invest in risk-free assets.
   - The calculations are complex and quite complicated.
2) Single Index Model:
   - There is an assumption that securities will only associate with one another, if these securities have the same response to market changes.
   - The return for each asset is calculated with the market return .
   - Simplifies complex Markowitz model calculations.

In doing return calculations in the Single Index Model include two element principal (Suteja & Gunardi, 2016) :

1) return element related to the company's uniqueness is symbolized by alpha (α).
2) return element related to the market is symbolized by beta (β).

This beta component has an important role in the single index model , because the level of sensitivity of security returns to market returns is symbolized with beta . The size mark expected returns For securities individual can be estimated using a single index model . Investors can also use assets free risk in his portfolio with use single index model (Qur'anitasari et al., 2016) . Systematically the single index model can be formulated as follows :

$$\mathrm{Ri} = a_i + \beta_i \mathrm{Rm} + e_i$$

Information:

Ri = Return of security i

a_i = The portion of return on security i that is not influenced by market performance

β_i = A measure of the sensitivity of security i's return to changes in market returns

Rm = Return market index r

e_i = Residual error

Suteja & Gunardi (2016) also mentioned in the Single Index Model Here, pay attention to several assumptions, namely :

1) ei is uncorrelated with ej for all values of i and j
2) ei is not correlated with market index returns

Based on analysis above can determined different or whether there is return and risk in selection shares and determination the optimal portfolio on the Indonesian Stock Exchange is reflected from high shares.

Possible hypothesis submitted in study this that is:

Hypothesis 1:

H0: There is no difference in portfolio returns using the Markowitz Model and Single Index Model .

Ha: There is a difference in portfolio returns using the Markowitz Model and the Single Index Model .

Hypothesis 2:

H0: There is no difference in portfolio risk using the Markowitz Model and Single Index Model.

Ha: There is a difference in portfolio risk using the Markowitz Model and the Single Index Model.

2. Research Methods

In this research, we use method purposive sampling with criteria ie share company registered and active traded on the consistent Indonesian Stock Exchange classified in LQ45 index for 2019 – 2023. Based on the criteria above so obtained a total of 27 samples.

Table 2. Research Samples

No.	Stock code	Company name
1.	ADRO	Adaro Energy Tbk
2.	AKRA	AKR Corporindo Tbk
3.	ANTM	Aneka Tambang (Persero) Tbk
4.	ASII	Astra International Tbk
5.	BBCA	Bank Central Asia Tbk
6.	BBNI	Bank Negara Indonesia (Persero) Tbk
7.	BBTN	State Savings Bank (Persero) Tbk
8.	BMRI	Bank Mandiri (Persero) Tbk
9.	BSDE	Bumi Serpong Damai Tbk
10.	CPIN	Charoen Pokphand Indonesia Tbk
11.	EXCL	XL Axiata Tbk
12.	GGRM	Gudang Garam Tbk
13.	HMSP	HM Sampoerna Tbk
14.	ICBP	Indofood CBP Sukses Makmur Tbk
15.	INDF	Indofood Sukses Makmur Tbk
16.	INTP	Indocement Tunggal Prakarsa Tbk
17.	JSMR	Jasa Marga (Persero) Tbk
18.	KLBF	Kalbe Farma Tbk
19.	MNCN	Media Nusantara Citra Tbk
20.	PGAS	Perusahaan Gas Negara (Persero) Tbk
21.	PTBA	Bukit Asam Coal Mine Tbk
22.	PWON	Pakuwon Jati Tbk
23.	SMGR	Semen Indonesia (Persero) Tbk
24.	TLKM	Telekomunikasi Indonesia (Persero) Tbk
25.	UNTR	United Tractors Tbk
26.	UNVR	Unilever Indonesia Tbk
27.	WIKA	Wijaya Karya (Persero) Tbk

a. Markowitz Model Analysis

First stage is analyzing the data obtained from the Indonesian Stock Exchange with use Markowitz Model (Markowitz, 1952), for determine portfolio following table the steps.

Table 3. Markowitz Model Data Analysis

No.	Information	Formula
1.	Count Stock Return Value (Ri)	$Ri = \frac{Pt-Pt-1}{Pt-1}$
2.	Calculating Expected Return Value (E(Ri))	$E(Ri) = \frac{\Sigma Ri}{n}$
3.	Calculating Stock Risk ($\sigma^2 i$)	$\sigma^2 i = \frac{\sqrt{\Sigma(Rit-E(Ri))^2}}{(n-1)}$
4.	Count Covariance Between Two Pieces Share (σ_{ij})	$\sigma_{ij} = \frac{[(Rit-E(Ri))(Rjt-E(Rj))]}{n}$
5.	Calculating Expected Return Portfolio $^{(E(Rp))}$	$E(Rp) = \Sigma WiE(Ri) + WjE(Rj)$
6.	Calculating Portfolio Risk ($\sigma^2 p$)	$\sigma^2 p = \sqrt{\Sigma WiWj\sigma ij}$

Source : (Hartono, 2020)

b. Single Index Model Analysis

After doing analysis optimal portfolio use Markowitz Model, next researcher will analyze use Single Index Model, meanwhile analysis presented in the following table.

Table 4. Single Index Model Data Analysis

No.	Information	Formula
1.	Count Mark Return Realization (Ri)	$Ri = \frac{Pt-Pt-1}{Pt-1}$
2.	Calculating Expected Return Value (E(Ri))	$E(Ri) = \frac{\Sigma Ri}{n}$
3.	Calculating Realized Market Return Value (Rm)	$Rm = \frac{\text{Indeks pasar}-\text{Indeks pasar t}-1}{\text{Indeks pasar t}-1}$
4.	Expected Return (E(Rm))	$E(Rm) = \frac{\Sigma Rm}{n}$
5.	Calculating Variance Return Individual ($\sigma^2 i$)	$\sigma^2 i = \Sigma \frac{(Ri-E(Ri))^2}{n}$
6.	Count Market Return Variance ($\sigma^2 m$)	$\sigma^2 m = \Sigma \frac{(Rm-E(Rm))^2}{n}$
7.	Count Covariance Ri and Fr (σim)	$\sigma_{im} = \Sigma \frac{[(Ri-E(Ri))(Rm-E(Rm))]}{n}$
8.	Count Beta (β i)	$\beta i = \frac{\sigma_{im}}{\sigma^2 m}$
9.	Count Alpha (αi)	$a_i = E(Ri) - (\beta i\, E(Rm))$
10.	Calculating Residual Variance Error/Unsystematic Risk ($\sigma^2 ei$)	$\sigma^2 ei = \beta^2 \sigma^2 m + \sigma^2 i$
11.	Calculating the Rate of Return Free Risk (Rf)	$R_f = \frac{\Sigma Rf}{12}$
12.	Calculating Excess Return to Beta (ERB)	$ERB = \frac{E(Ri)-Rf}{\beta_i}$
13.	Count Mark A_i	$Ai = \frac{[E(R_i)-Rf]\beta_i}{\sigma^2 ei}$
14.	Calculating Bi Values	$B_i = \frac{\beta i^2}{\sigma^2 ei}$
15.	Count Ci And Cut- Off Points (C*)	$C_i = \frac{\sigma^2 m[Ai]}{1+\sigma^2 m[Bi]}$
16.	Calculating Fund Proportion (Wi)	$W_i = \frac{Z_i}{\Sigma Zj}$ and $Z_i = \frac{\beta_i}{\sigma^2 ei}(ERB - C*)$
17.	Count Beta Portfolio (β p)	$\beta_P = \Sigma W_i \beta_i$
18.	Count Alpha Portfolio (αp)	$\alpha_P = \Sigma W_i \alpha_i$
19.	Count Unsystematic Risk Portfolio ($\sigma^2 ep$)	$\sigma^2 ep = \Sigma W_i \sigma^2 ei$
20.	Count Portfolio Expected Return Rate Optimal (E(Rp))	$E(R_{p)} = \alpha p + \beta p. E(R_{m)}$
21.	Calculating Levels Risk Optimal Portfolio ($\sigma^2 p$)	$\sigma^2 p = \beta\rho^2 \sigma^2 m + \sigma^2 ep$

Source : (Hartono, 2020)

c. Test Assumptions Classic

Test assumptions classic aim For know what is the equation model regression truly show significant and representative relationship. Testing this is also purposeful For ensure that regression model the worthy or No For done testing furthermore.

d. Normality test

Normality test that is with the Kolmogorov-Smirnov test . Testing This use method Kolmogorov-Smirnov with criteria testing $\alpha = 0.05$ as following:

1) If α sig ≥ α means sample data normally distributed.
2) If α sig ≤ α means sample data No normally distributed.

e. Homogenety Test

According to (Nuryadi et al., 2017) homogeneity test is something intended statistical test procedures For show that two or more sample data group originate from population that has the same variance. On analysis regression, requirements analysis required is tool regression for every grouping based on variable bound own the same variance. So, you can said that homogeneity test aim For look for know is from a number of research data group own the same variance or no. In other words, homogeneity means that our data set thorough own the same characteristics.

f. Test the Difference

Hypothesis testing carried out in study This using different tests independent sample t-test and Mann Whitney. Test differently This done with condition moreover formerly carry out normality and homogeneity tests. Testing This using SPSS 23 software.

Difference Test t-test

This method aim For test is variance population second sample The same or No with use Levene test for equality of variance and measures the test as following:

1. Determine Hypothesis

Hypothesis 1:
Ho = Return Portfolio use Markowitz Model and Single Index Model is the same.
Ha ≠ Return Portfolio use Markowitz Model and Single Index Model is not the same.
Hypothesis 2:
Ho = Risk Portfolio use Markowitz Model and Single Index Model is the same.
Ha ≠ Risk Portfolio use Markowitz Model and Single Index Model is not the same.

2. Decision Criteria

If significance is > 5% then Ho is accepted, Ha is rejected
If significance < 5% then Ho is rejected, Ha is accepted

g. Mann Whitney Difference Test

Mann Whitney is an equivalent non - parametric test with the t test, however possible there is difference amount studied samples. Uses of the Mann test Whitney that is for test difference second group independent when the data is not fulfil condition For tested with t-test procedure. Test steps as following:

1) Determine Tested hypothesis

Hypothesis 1:
Ho = Portfolio Return use Markowitz Model and Single Index Model is the same.
Ha ≠ Portfolio Return use Markowitz Model and Single Index Model is not the same.
Hypothesis 2:
Ho = Risk Portfolio use Markowitz Model and Single Index Model is the same.
Ha ≠ Risk Portfolio use Markowitz Model and Single Index Model is not the same.

2) Determine level of significance (α)

The level of significance (α) is used that is $\alpha = 0.05$

3) Criteria decision

If significance is > 5% then Ho is accepted, Ha is rejected.
If significance < 5% then Ho is rejected, Ha is accepted.

3. Results and Discussion

3.1. Markowitz Model Analysis

Composition, magnitude proportion optimal portfolio of shares listed on the LQ45 index, and valuation performance according to Markowitz Model can is known with method do various step as following:

a. The calculation results return and expected return of each share

Owning shares expected return negative no included in calculation furthermore because if results expected return negative later will produce expected return negative portfolio too. Expected return negative No including portfolio efficient, so no possible for made candidate optimal portfolio. Table 5 follows This show calculation expected return during period study.

Table 5. Expected Stock Return

No.	Issuer	Company name	(E(Ri)
1	ADRO	Adaro Energy Tbk	0.0313
2	AKRA	AKR Corporindo Tbk	0.0172
3	ANTM	Aneka Tambang (Persero) Tbk	0.0342
4	ASII	Astra International Tbk	-0.003
5	BBCA	Bank Central Asia Tbk	0.012

No.	Issuer	Company name	(E(Ri)
6	BBNI	Bank Negara Indonesia (Persero) Tbk	0.0084
7	BBTN	State Savings Bank (Persero) Tbk	0.0001
8	BMRI	Bank Mandiri (Persero) Tbk	0.0103
9	BSDE	Bumi Serpong Damai Tbk	-0.0018
10	CPIN	Charoen Pokphand Indonesia Tbk	-0.0015
11	EXCL	XL Axiata Tbk	0.0073
12	GGRM	Gudang Garam Tbk	-0.0269
13	HMSP	HM Sampoerna Tbk	-0.0271
14	ICBP	Indofood CBP Sukses Makmur Tbk	0.0013
15	INDF	Indofood Sukses Makmur Tbk	-0.0000
16	INTP	Indocement Tunggal Prakarsa Tbk	-0.0087
17	JSMR	Jasa Marga (Persero) Tbk	-0.0004
18	KLBF	Kalbe Farma Tbk	0.0087
19	MNCN	Media Nusantara Citra Tbk	0.0104
20	PGAS	Perusahaan Gas Negara (Persero) Tbk	0.0069
21	PTBA	Bukit Asam Coal Mine Tbk	0.0157
22	PWON	Pakuwon Jati Tbk	-0.0001
23	SMGR	Semen Indonesia (Persero) Tbk	-0.0049
24	TLKM	Telekomunikasi Indonesia (Persero) Tbk	0.0022
25	UNTR	United Tractors Tbk	0.0037
26	UNVR	Unilever Indonesia Tbk	-0.0111
27	WIKA	Wijaya Karya (Persero) Tbk	-0.0035

Source : Processed data , 2024

Based on Table 5, results calculation of 27 shares there are 12 shares owned negative expected return. The calculation results have expected return negative No included in calculation next. It means owning shares expected return positive there are 15 shares candidate portfolio efficient.

Table 6. Stocks that Have Positive Expected Returns

No.	Issuer	Company name	E(Ri)
1	ADRO	Adaro Energy Tbk	0.0313
2	AKRA	AKR Corporindo Tbk	0.0172
3	ANTM	Aneka Tambang (Persero) Tbk	0.0342
4	BBCA	Bank Central Asia Tbk	0.012
5	BBNI	Bank Negara Indonesia (Persero) Tbk	0.0084
6	BBTN	State Savings Bank (Persero) Tbk	0.0001
7	BMRI	Bank Mandiri (Persero) Tbk	0.0103
8	EXCL	XL Axiata Tbk	0.0073
9	ICBP	Indofood CBP Sukses Makmur Tbk	0.0013
10	KLBF	Kalbe Farma Tbk	0.0087
11	MNCN	Media Nusantara Citra Tbk	0.0104
12	PGAS	Perusahaan Gas Negara (Persero) Tbk	0.0069
13	PTBA	Bukit Asam Coal Mine Tbk	0.0157
14	TLKM	Telekomunikasi Indonesia (Persero) Tbk	0.0022
15	UNTR	United Tractors Tbk	0.0037

b. The calculation results risk of each stock

Risk describe possibility deviation returns realization and return expectation. Return and risk tend move simultaneously, which means owning shares high returns will tend own high risk too.

Table 7. Standard Individual Stock Deviations and Variances

No.	Issuer	σ_i	σ_i^2
1	ADRO	0.123	0.0151
2	AKRA	0.1181	0.0139
3	ANTM	0.1779	0.0316
4	BBCA	0.0565	0.0032
5	BBNI	0.1154	0.0133
6	BBTN	0.1621	0.0263
7	BMRI	0.0865	0.0075
8	EXCL	0.1061	0.0113
9	ICBP	0.0671	0.0045
10	KLBF	0.0644	0.0042
11	MNCN	0.1349	0.0182
12	PGAS	0.144	0.0207
13	PTBA	0.2181	0.0476
14	TLKM	0.0667	0.0045
15	UNTR	0.0995	0.0099

Source : Processed data , 2024

c. The calculation results proportion share use application Solver on Ms. Excel

Table 8. Optimal Solver Value Portfolio Share Proportion

No.	Issuer	Solver Value
1.	ADRO	0.49
2.	AKRA	-
3.	ANTM	0.22
4.	BBCA	0.07
5.	BBNI	-
6.	BBTN	-
7.	BMRI	-
8.	EXCL	-
9.	ICBP	0.05
10.	KLBF	0.16
11.	MNCN	-
12.	PGAS	-
13.	PTBA	-
14.	TLKM	-
15.	UNTR	-

Source : Processed data , 2024

Based on Table 8 which is results calculations on the solver application, shows that of 15 shares candidate shaper optimal portfolio with Markowitz Model is ADRO with value 0.49 or 49%, ANTM with value 0.22 or 22%, BBCA with value 0.07 or 7%, ICBP with value 0.05% or 5%, and KLBF with value 0.16 or 16%. Meanwhile, 10 shares the rest No including into the optimal portfolio.

d. The calculation results Return Portfolio, Risk portfolio and Optimal Portfolio Performance using Markowitz Model

Table 9. Portfolio Return, Risk Portfolio and Optimal Portfolio Performance using the Markowitz Model

Information	Mark
Portfolio Return	0.025382
Portfolio Risk	0.081342
Sharpe Ratio	0.2678
Treynor Ratio	0.0148
Jensen Alpha Ratio	0.0307

Source : Processed data , 2024

Return value portfolio based on Markowitz Model obtained of 0.025382 or 2.54% and value risk portfolio amounting to 0.081342 or 8.13%. With mark Sharpe Ratio of 0.2678, value Treynor Ratio of 0.0148, and the Jensen Alpha Ratio of 0.0307.

3.2. Single Index Model Analysis

Composition, magnitude proportion optimal portfolio of shares listed on the LQ45 index, and valuation performance according to Single Index Model can is known with method do various step as following :

a) The calculation results returns share

Shares created sample like sample on the Markowitz Model that is owning shares expected return positive shown in table 6. Amount owning shares expected return positive namely 15 shares.

b) The calculation results market return and market risk

Market return (R_m) is calculated by measuring the difference between the IHSG in the current month ($IHSG_t$) and the IHSG in the previous month ($IHSG_{t-1}$) and then shared with the IHSG month previously (IHSGt-1). expected return (E(R_m)) is calculated based on the average percentage of IHSG returns divided with amount returns IHSG index. Based on table 4.10, the IHSG expected return is 0.0031 (0.31%) per month and the market variance is 0.00187 (0.19%).

Table 10. Expected Return, Variance, and Standard IHSG Deviation

E(Rm)	0.0031
$\sigma_{rm}{}^2$	0.00187
Standard Deviation	0.0433

Source : Processed data , 2024

c) The calculation results Beta and Alpha of each stock

Beta is risk unique from individual shares and use for count Excess Return to Beta (ERB). The more big mark beta, then the more the risk is great systematically. Beta is sensitivity returns share to market returns. Beta positive indicated that if market returns increases, then returns shares will too increase. On the contrary if beta negative, increase market returns will followed decline returns share. Beta calculated with compare covariance share with market variance.

Alpha is mark expectation from returns independent securities to the market, so alpha that has mark positive will can add returns independent expectations to market returns (Ratna et al., 2017.) . Alpha calculated with subtract expected returns share with beta product with expected return market.

Table 11. Beta and Alpha of Each Stock

No.	Issuer	β	α
1	ADRO	1.2147	0.0276
2	AKRA	1.9121	0.0114
3	ANTM	2.7847	0.0257
4	BBCA	0.921	0.0092
5	BBNI	2.0957	0.002
6	BBTN	2,547	-0.0077
7	BMRI	1.4256	0.0059
8	EXCL	1,283	0.0033
9	ICBP	-0.0191	0.0014
10	KLBF	0.5117	0.0071
11	MNCN	1.7375	0.0051

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No.	Issuer	β	α
12	PGAS	2.7527	-0.0016
13	PTBA	1,052	0.0125
14	TLKM	0.9817	-0.0008
15	UNTR	0.8247	0.0012

Source : Processed data , 2024

Based on Table 11, there are owning shares mark beta more from one (1) viz ADRO, AKRA, ANTM, BBNI, BBTN, BMRI, EXCL, MNCN, PGAS and PTBA shares. ANTM shares as owning shares mark beta highest amounting to 2.7847. It means if market returns increase one by one, then There is enhancement returns ANTM shares amounted to 2.7847 units. According to (Samsul, 2015) , value beta more from one (β i >1) means that risk systematic share more big compared to with risk market systematics . Beta worth not enough from one (β i < 1) indicates risk systematic share more small compared to with risk systematic markets, whereas if beta worth one (βi =1) then risk systematic share will the same with risk market systematics.

d) The calculation results risk investment

(Tandelilin, 2017) say that risk is possibility differences that occur between returns actual with returns hope. The more big possibility the difference is meaningful the more big risk investment the. Risk calculated investment consists from variant from residual error (σ_{ei}^2), variance market return (σ_{rm}^2), and risk shares (σ_i^2). Following is variant data from residual error (σ_{ei}^2), market return variance (σ_{rm}^2), and risk shares (σ_i^2).

Table 12. Residual Error Variance, Market Variance and Stock Variance

No.	Issuer	σ_{ei}^2	σ_{rm}^2	σ_i^2
1	ADRO	0.0179	0.00187	0.0151
2	AKRA	0.0208	0.00187	0.0139
3	ANTM	0.0462	0.00187	0.0316
4	BBCA	0.0048	0.00187	0.0032
5	BBNI	0.0215	0.00187	0.0133
6	BBTN	0.0384	0.00187	0.0263
7	BMRI	0.0113	0.00187	0.0075
8	EXCL	0.0143	0.00187	0.0113
9	ICBP	0.0045	0.00187	0.0045
10	KLBF	0.0046	0.00187	0.0042
11	MNCN	0.0238	0.00187	0.0182
12	PGAS	0.0349	0.00187	0.0207
13	PTBA	0.0496	0.00187	0.0476
14	TLKM	0.0063	0.00187	0.0045
15	UNTR	0.0112	0.00187	0.0099

Source : Processed data , 2024

e) The calculation results Excess Return to Beta (ERB)

Excess Return to Beta measure excess returns relatively to one unit of risk that does not can measured diversification with beta (Hartono, 2020) . The ERB ratio shows connection two factor decider investment that is return and risk. Stocks included in composition the optimal portfolio is owning shares mark excess return to beta (ERB) is more big compared to with mark cut-off point (Ci). The calculation results ERB values are sorted from the biggest down to the smallest.

Table 13. ERB Value Calculation Results

No.	Issuer	ERB
1	ICBP	2.4352
2	UNTR	0.0755
3	BBCA	0.0649
4	PTBA	0.0622
5	EXCL	0.0375
6	MNCN	0.0326
7	ADRO	0.0228
8	BBTN	0.0202
9	ANTM	0.0164
10	AKRA	-0.0055
11	PGAS	-0.0181
12	BBNI	-0.0245
13	BMRI	-0.0288
14	TLKM	-0.0596
15	KLBF	-0.0904

Source : Processed data , 2024

d. The calculation results mark Ai, Bi, and Ci

Ai and Bi values calculated for get mark Aj and Bj required for count mark cut off point (C*). Owning shares Ai highest is BBCA of 11,523, while the lowest is ICBP of 0.0013. Owning shares Bi highest is PGAS of 216.9361, while the lowest is ICBP of 0.0813.

Table 14. Ai, Bi and Ci Values for Each Share

No.	ISSUER	Ai	Bi	Ci
1	ADRO	1.8804	82.4363	0.003
2	AKRA	-0.9627	175.9	-0.0014
3	ANTM	2.6971	167.9963	0.0038
4	BBCA	11,523	177.5558	0.0162
5	BBNI	-4.9954	203.9001	-0.0068
6	BBTN	3.4108	168.8513	0.0049
7	BMRI	-5.1926	179.9994	-0.0073
8	EXCL	4.3076	114,784	0.0066
9	ICBP	0.0013	0.0813	0.0004
10	KLBF	-5,095	56.3877	-0.0086
11	MNCN	4.1267	126.5997	0.0062
12	PGAS	-3.9203	216.9361	-0.0052
13	PTBA	1.3879	22.2981	0.0025
14	TLKM	-9.1818	154.0166	-0.0133
15	UNTR	4.5979	60.9136	0.0077

Source : Processed data , 2024

f) The calculation results cut-off point (C*)
Determine cut-off point (C*) for determine optimal portfolio. The size cut-off point (C*) is mark Ci Where ERB value last time still more big from mark C*. The grade C* is used for determine point barrier which shares are included as candidate optimal portfolio.

Table 15. List of Stocks that Have an ERB Greater than C*

No.	Issuer	ERB	C*	Information
1	ICBP	2.4352	0.0162	OPTIMAL
2	UNTR	0.0755	0.0162	OPTIMAL
3	BBCA	0.0649	0.0162	OPTIMAL
4	PTBA	0.0622	0.0162	OPTIMAL
5	EXCL	0.0375	0.0162	OPTIMAL
6	MNCN	0.0326	0.0162	OPTIMAL
7	ADRO	0.0228	0.0162	OPTIMAL
8	BBTN	0.0202	0.0162	OPTIMAL
9	ANTM	0.0164	0.0162	OPTIMAL

Source : Processed data , 2024

Based on table 15, there are 9 shares included in optimal portfolio because own higher ERB value big from C* that is ICBP (Indofood CBP Sukses Makmur Tbk), UNTR (United Tractors Tbk), BBCA (Bank Central Asia Tbk), PTBA (Tambang Batubara Bukit Asam Tbk), EXCL (XL Axiata Tbk), MNCN (Media Nusantara Citra Tbk), ADRO (Adaro Energy Tbk), BBTN (State Savings Bank (Persero) Tbk), and ANTM (Aneka Tambang (Persero) Tbk).

g) The calculation results scale weighted (Zi) and large proportion of funds (Wi) for each share.
Count proportion of funds (Wi) is carried out after count scale weighted.

Table 17. Calculation of Weighted Scale (Zi) and Fund Proportion (Wi)

No.	Issuer	Z_i	W_i
1	UNTR	4.38	15.18%
2	PTBA	0.98	3.38%
3	MNCN	1.20	4.15%
4	ICBP	10.27	35.59%
5	EXCL	1.91	6.62%
6	BBTN	0.27	0.92%
7	BBCA	9.39	32.55%
8	ANTM	0.01	0.04%
9	ADRO	0.45	1.56%
	Amount	28.85	100%

Source : Processed data , 2024

e. The calculation results Return Portfolio, Risk portfolio and Optimal Portfolio Performance using Single Index Model

Table 18. Portfolio Returns, Risk Portfolio, and Optimal Portfolio Performance using the Single Index Model

Information	Mark
Portfolio Return	0.02069
Portfolio Risk	0.04005
Sharpe Ratio	0.42540
Treynor Ratio	0.00734
Jensen Alpha Ratio	0.01353

Source : Processed data , 2024

Return value portfolio based on Single Index Model obtained of 0.02066 or 2.07% and value risk portfolio of 0.04005 or 4%. With mark Sharpe Ratio of 0.42540, value Treynor Ratio of 0.00734, and Jensen Alpha Ratio as big as 0.01353.

3.3. Test the Difference

Table 19. Independent Sample T-test Return Difference Test Results
Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Return	Equal variances assumed	,568	,466	,631	12	,540	.0046111	.0073098	-.0113156	.0205379
	Equal variances not assumed			,601	7,292	,566	.0046111	.0076772	-.0133964	.0226186

Source : SPSS Output , 2024

Based on statistical data tests in table 19, the test results are different independent samples t-test with level 95% confidence shows that mark Sig.(2-tailed) own mark as big as 0.540 > 0.05 so Ho is accepted and Ha is rejected, which means returns portfolio use Markowitz Model and Single Index Model is the same.

Table 20. Risk Difference Test Results using Independent Sample T-test

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Risk	Equal variances assumed	,007	,936	-1,011	12	,332	-.0294644	.0291460	-.0929681	.0340392
	Equal variances not assumed			-1.013	8,441	,339	-.0294644	.0290795	-.0959176	.0369887

Source : SPSS Output , 2024

Based on statistical data tests in table 20, the test results are different independent sample t-test with level 95% confidence shows that mark Sig.(2-tailed) own mark equal to 0.332 > 0.05 so Ho is accepted and Ha is rejected, which means risk portfolio use Markowitz Model and Single Index Model is the same.

4. Conclusion

Based on results analysis comparison formation optimal portfolio between Markowitz Model and Single Index Model have been done, then can obtained a number of conclusions as followin: Composition share results formation optimal portfolio use Markowitz Model consists of 5 shares that is shares of ADRO (Adaro Energy Tbk), ANTM (Aneka Tambang Persero Tbk.), BBCA (Bank Central Asia Tbk.), ICBP (Indofood CBP Sukses Makmur Tbk.), and KLBF (Kalbe Farma Tbk).

Composition share results formation optimal portfolio use The Single Index Model consists of of 9 shares that is shares of ICBP (Indofood CBP Sukses Makmur Tbk.), UNTR (United Tractors Tbk), BBCA (Bank Central Asia Tbk), PTBA (Tambang Batubara Bukit Asam Tbk), EXCL (XL Axiata Tbk), MNCN (Media Nusantara Citra Tbk), ADRO (Adaro Energy Tbk), BBTN (Bank Tabungan Negara (Persero) Tbk), and ANTM (Aneka Tambang (Persero) Tbk).

Based on 5 shares entered in optimal portfolio use Markowitz Model the expected have returns amounting to 0.025382 or 2.54% per month and the risk must be faced by top investors investment in 5 shares the is amounting to 0.081342 or 8.13%. Whereas based on 9 shares entered in optimal portfolio use The Single Index Model expected have returns amounting to 0.02066 or 2.1% per month and the risk must be faced by top investors investment in 9 shares the is of 0.04005 or 4%. The risks contained in this optimal portfolio more small compared to with risk if investing in individual stocks. Formation optimal portfolio is one method diversification for reduce risk.

Proportion of funds allocated for investment share results formation Markowitz Model optimal portfolio is ADRO shares are 49%, ANTM is 22%, BBCA is 7%, ICBP is 5%, and KLBF is 16%. Whereas proportion of funds allocated for investment share results formation the optimal portfolio of the Single Index Model is UNTR shares are 15.18%, PTBA is 3.38%, MNCN is 4.15%, ICBP is 35.59%, EXCL is 6.62%, BBTN is 0.92%, BBCA is 32.55%, ANTM is 0.04% and ADRO is 1.56%. Markowitz Model optimal portfolio performance use Sharpe Ratio of 0.2678, using Treynor Ratio of 0.0148, and use Jensen Alpha Ratio of 0.0307. Meanwhile, in the Single Index Model, performance optimal portfolio use Sharpe Ratio of 0.42540, using Treynor Ratio of 0.00734, and Jensen Alpha Ratio of 0.01353. Based on calculating the Sharpe Ratio, Treynor Ratio and Jensen Alpha Ratio is a good model its performance in formation optimal portfolio ie Markowitz Model Because mark from Sharpe Ratio and Jensen Alpha Ratio more tall compared to Single Index Model.

The findings suggest several key implications for investors, financial practitioners, researchers, and the capital market. For investors, the Markowitz Model offers higher potential returns but with greater risk, making it suitable for those with a higher risk appetite, while the Single Index Model provides lower risk, appealing to more conservative investors. Investment managers can adopt a flexible approach by combining both models to balance return and risk based on client needs. Academically, this research encourages further exploration into hybrid models that integrate the strengths of both approaches and highlights the importance of performance evaluation using metrics such as Sharpe Ratio, Treynor Ratio, and Jensen Alpha.

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